CCSU Regional Math Competition, 2014

Part I

Each problem is worth ten points. Please be sure to use separate pages to write your solution for every problem.

Problem 1. You are floating down the middle of a river. It is 1000 feet wide and flows at 11 feet per second. Suddenly you notice a waterfall 230 feet ahead. Unfortunately, you can only swim 10 feet per second in still water. Can you reach the bank before being swept over the falls?

Problem 2. We are given 2015 positive integers. We know that if we take away any one of them, the remaining 2014 integers can be partitioned into two sets with the same number of elements and the same sum of elements. Show that all integers must be equal.

Problem 3. Let P be the parabola $y = x^2$. Let A be any point on P other than the vertex. Let L be the line orthogonal to the tangent line to P at A. Let B be the other point at which L crosses P. Find the smallest possible area of the bounded region lying between P and the segment AB.

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Part II

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Problem 4. Show that there is a point A on the surface S of a cube of side 1 that can be joined with any other point on S by a piecewise straight line path contained in S of length at most 2.

Problem 5. Let $x_1, x_2, ..., x_{49}, x_{50}$ be 50 real numbers, not all equal. The mean μ and the standard deviation σ are given by

$$\mu = \frac{1}{50}\sum_{i=1}^{50} x_i, \qquad \sigma = \sqrt{\frac{1}{50}\sum_{i=1}^{50}(x_i - \mu)^2}.$$

The z-score for a particular value x_k is given by $z = \dfrac{x_k - \mu}{\sigma}$. It measures the distance of x_k from the mean in standardized units of σ. Find the largest possible value of the z-score for x_1 and show why it is the largest possible.

Problem 6. Given $C > 0$, find all non-negative continuous functions f defined on $[0, \infty)$, which satisfy the following inequality for all $x \geq 0$

$$f(x) \leq C \cdot \int_0^x f(t)dt.$$