

CCSU Regional Math Competition, 2019

Part I

Each problem is worth ten points. Please be sure to use separate pages to write your solution for every problem.

Problem 1. Show that in each year there is a month whose first day is a Sunday and a month whose second day is a Saturday.

Problem 2. Find the largest natural number n such that $n + 3$ divides $n^4 + 2019$.

Problem 3. Let P be a parabola in the x - y plane having the following properties: the axis of P is parallel to the y -axis, the vertex of P lies on the segment with endpoints $(1, 0)$ and $(0, 1)$, and P passes through the origin. Let R be the bounded region between P and the x -axis. What is the largest possible area of R ?

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Part II

Each problem is worth ten points. Please be sure to use separate pages to write your solution for every problem.

Problem 4. Let P be a point inside a regular hexagon with side length 1. Suppose the distance from P to vertex Q is $13/12$ and the distance from P to vertex R is $5/12$.

- a) Show that Q and R are adjacent vertices.
- b) Suppose S is the other vertex adjacent to R (i.e. $S \neq Q$). Find the distance from P to S .

Problem 5. Let L be a language specified as follows. The alphabet for L is A,B,C,D,E,F. A word in L is any (non-empty) string of letters, provided that the letters occur in alphabetical order and no letter occurs more than once. A sentence in L is any string of words, provided that the words occur in dictionary order, each letter of the alphabet appears in the sentence exactly once, and no two words in the sentence have the same length. (For example, BAD is not a word in L because its letters do not occur in alphabetical order; CF ABDE is not a sentence in L because the two words do not occur in dictionary order.) Find the total number of three-word sentences in L .

Problem 6. Suppose it is snowing at a constant rate (say in inches per hour) and that a snowplow is out plowing snow at a constant rate (in cubic feet per second). During the first hour of plowing the snowplow traveled twice as far as it did during the second hour of plowing. Assume that the snowplow travels in a straight line and is always plowing unplowed snow. For how many hours had it been snowing before the snowplow started plowing?